g. No. : .....

me : .....

Third Semester B.Sc. Degree Examination, December 2016
First Degree Programme Under CBCSS
COMPLEMENTARY COURSE FOR PHYSICS
MM 1331.1 : Differential Equations, Theory of Equations and Theory of Matrices
(2014 Admission Onwards)

me : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Define the degree and order of a differential equation.
2. Give an example of homogeneous first order ordinary differential equation.
3. Write a necessary condition for an exact differential equation.
4. Define the angle between any two curves.
5. Define the rank of a matrix.
6. Write any two elementary transformation for matrices.
7. Define eigen value of a matrix.
8. State fundamental theorem of algebra.
9. If $\sqrt{p} + \sqrt{q}$ is a root of a fourth degree equation find the other three roots.
10. Define a singular matrix.

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SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2 marks each**.

11. Solve $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$.

12. Solve $(x^2 + y)dx + (y^2 + x)dy = 0$.

13. Solve $y'' + 2y' + 5y = 0$.

14. Solve $y'' + 4y' + 4y = \cos 5x$.

15. Find the characteristic polynomial of $A = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$.

16. Solve the system of equations $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$

17. Find the rank of $A = \begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \end{pmatrix}$ by reducing to echelon form.

18. Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & -1 & 3 \end{pmatrix}$.

19. Two roots of the equation $x^4 - 6x^3 + 18x^2 - 30x + 25 = 0$ are of the form $\alpha +$ and $\beta + i\alpha$. Find all roots of the equation.

20. Show that the equation $12x^2 - x^4 + 10x^3 - 28 = 0$ has atleast four imagina roots.

21. Given that the roots of the equation $4x^3 - 24x^2 + 23x + 18 = 0$ are in arithme progression. Solve the equation completely.

22. Explain any two methods to find approximate roots of an equation.

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SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Solve $x^2y'' - 5xy' + 5y = 0$.

24. Find the differential equation of the family of parabolas $y = cx^2$ and find its orthogonal trajectory.

25. Solve $y''' - 2y'' - y' + 2y = 0$.

26. Show that $A = \begin{pmatrix} 9 & 4 & 4 \\ -8 & 3 & 4 \\ 16 & 8 & 7 \end{pmatrix}$ is diagonalizable.

27. If $A = \begin{pmatrix} 2 & 3 & 5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$ show that $A^2 = A$.

28. Show that the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear if and only if the rank of the matrix $\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$ is less than three.

29. Solve $x^3 + x^2 + x - 100 = 0$ by Newton-Raphson method correct to two decimal places.

30. If the sum of the two roots of the equation $4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$ be zero find all the roots of the equation.

31. Show by an example that $AX = 0$ has the same solutions as the reduced system $A_RX = 0$.

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SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Solve $(x - y)^2 \frac{dy}{dx} = a^2$.

33. Find the eigen values and corresponding eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

34. Discuss without solving the following systems of equations have a unique solution or not

    i) $x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1$.

    ii) $x + 2y + 3z = 0$, $2x + 3y + 4z = 0$, $7x + 13y + 19z = 0$.

35. State Descartes' rule of signs and apply it to prove the equation $x^3 + 2x + 3 = 0$ has one negative and two imaginary roots.

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